Assignment / Due Date:

For your sub-system, choose one member in tension or one member in compression. Choose white pine as your material. Make appropriate assumptions and then do the relevant geometry, dead-weight, live load and member force calculations. Then do the appropriate maximum allowable stress calculation. Draw your conclusion. Due beginning of class two days after the date at the bottom of this page– drop it off for marking

Thinking: ____ / 100

Thinking Like Telford – Strength of Materials

Grade 12 Technological Design

CONTENTS

NOTE: First, be sure to:

- **Review file 8-Step_Structural_Analysis.doc**
- **Re-do the "*Modeling a Structure*" Moodle quizzes at http://thinkproblemsolving.org**
- **In the preceding work (and based on your informed assumptions), you calculated the force that is acting on a member of a structure – either in compression (a "push") or in tension (a "pull"). Next, you must determine whether or not the member can safely and effectively withstand that calculated force.**

1 How Does a Structural Member Respond to the Force Acting on It?

In Moodle quiz "Suspension Cable / Towers Free Body" on http://thinkproblemsolving.org there was found to be tension in the leftmost cable of 229 pounds force. This cable member – or even a piece of rope or a piece of wood or steel – must resist this internal tension force. The member must not break or permanently deform. The resistance to such changes is a property of the material. How can we be sure?

But first… the fundamentals…

1.1 Some Evolution of Our Understanding of Strength of Materials

1.1.1 Telford's Bedtime Reading

In Thomas Telford's time as an apprentice stonemason, the science of the strength of materials was not particularly well understood. The theory was certainly being developed by various scientists including Thomas Young (1773-1829). Young made several comments in his 1807 "*Course of Lectures on Natural Philosophy and the Mechanical Arts*" that are particularly significant in our efforts to "***Think Like Telford***" as we design an all-wood 36 foot-long model of his Menai Bridge.

With respect to lateral buckling of compressed columns, Young stated:

> *Considerable irregularities may be observed in all the experiments which have been made on the flexure of columns and rafters exposed to longitudinal forces; and there is no doubt but that some of them were occasioned by the difficulty of applying the force precisely at the extremities of the axis, and others by the accidental inequalities of the substances, of which the fibres must often have been is such directions as to constitute originally rather bent than straight columns.*

Young also introduced the idea that initial "yield" of a material is very important to avoid when loading a structural member:

> *A permanent alteration of form… limits the strength of materials with regard to practical purposes, almost as much as fracture, since in general the force which is capable of producing this effect is sufficient, with a small addition, to increase it till fracture takes place.*

Thomas Young is particularly remembered for his work in developing the idea of "Young's Modulus, E". E is a property of a material -- how much it will change in shape or length under a given stress.

$$E = \frac{Stress}{Strain}$$

E is a constant for a given material. Stress is force per unit area of the material. Strain is essentially "stretch" or change in length -- either shorter in the case of compression or longer in the case of tension. By way of simple example, the deflection of a spring is proportional to the load that is pulling on it.

Thomas Young was, indeed, well ahead of his time as he even surmised on the extremely small diameter of molecules which make up a material.[i]

We can imagine that civil engineer Thomas Telford probably read Young's papers and probably drew his own conclusions, which perhaps included:

1. Test methods must be extremely precise and any deviation in test methods and measurements must be documented and a reasonable attempt made to modestly understand the impact of such deviations
2. Samples of a test material may have not been quite "ideal" at the "macroscopic" level – that level which can be reasonably observed visually
3. Samples of a material under test may behave somewhat differently because they may be slightly different at the "small" – microscopic -- level for some reason
4. There is significant "uncertainty" in how a structure will respond to loads. The first sign of a distortion that is likely to be permanent must be avoided because complete fracture is very close at hand. Therefore, one must keep the applied loads well below those loads which may cause "*permanent alteration of form*" (aka plastic deformation). This is the notion of "factor of safety".
    a. **Note**: Factors of Safety in a particular industry reflect the degree of uncertainty in design / loading as well as the risk that the industry is willing to take vis a vis a failure and the possible consequences of the failure. In addition, careful attention to quality control throughout is key. Thomas Telford was on the leading edge of a new industry that used iron in structures – best practices were in their very early stages. Uncertainty was the order of the day -- it was evidently observed that Telford often prayed as he watched the chains on his suspension bridges first being loaded.

1.2 Materials for Bridges

Stone was the bridge-building material of choice for centuries – stone's huge mass put the entire sub-structure into compression. As long as the soil / rock conditions were constantly pushing inward on the bridge abutments, a well-built stone bridge could stand for centuries (subject to some upkeep of any mortar.) Stonemasons were among the most well-respected workers of that era. Wooden bridges were also built but, relative to stone, did not have near the durability and resistance to the environment.

In the late eighteenth century, engineers began to transition from stone to iron as their bridge-building material of choice. Stonemason / civil engineer Thomas Telford was another of the pioneers in experimentation regarding the strength of materials, especially iron. For example, refer to "*Papers Relating to the Building a Bridge Over the Menai Strait*" in the "*Repertory of Arts, Manufactures and Agriculture…*" Vol 35, Second Series, 1819. Blacksmiths and millwrights were the original mechanical engineers of the industrial revolution.

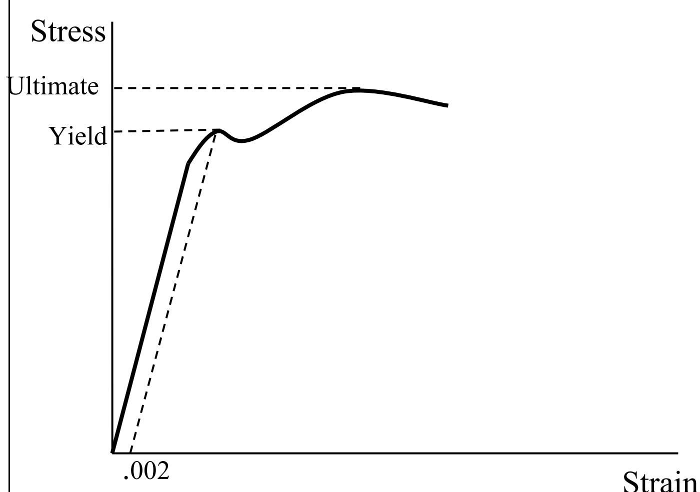
Telford was perhaps somewhat sceptical of some of the prevailing theory and the supporting mathematical formulas. He preferred to do his own testing in support of his designs. Telford did thousands of physical experiments on samples of material, keeping careful records of test data.

Telford was instrumental in setting up a foundation for the modern quality control industry. The data he collected on strength of materials served as part of a starting point for our modern "empirical" observation-based standards and codes.

1.3 Key Concepts and Measures of the Strength of a Material

Also, review "Structure_Concepts_Keywords.doc" in the pickup folder. Some of the concepts below relate more to metal than to general materials.

Keyword / Concept	An Initial Definition	Some "Things to Think About" Regarding Design and Fabrication Using Materials in General
Assumptions	The differences between reality and your model.	Modelling or simplifying a complex system depends on making reasonable assumptions. The assumptions must just not violate or obscure particularly important aspects of the system being modelled. Those aspects of the system that you want to preserve in your model are established through defined "requirements".
Brittleness	How quickly a material fractures as it is deformed by an applied load.	Window glass and cast iron are very brittle.
Ductility	A material's ability to be formed into shapes without cracking or fracturing.	Sheet steel is very ductile. Green wood is more ductile than bone-dry wood.
Hardness	A material's resistance to scratches and wear.	
I-Beam	A structural shape that looks like the letter "I". There are many classes of I-beam, each for particular applications / spans.	Some I-beams are cast. An I beam could also be fabricated by welding a top "flange" to the top edge of the "web" and the bottom flange to the bottom edge of the web. I-beams are used to resist bending loads -- material is placed in the I-beam where it is most needed to resist the stresses due to bending -- tension in one flange and compression in the opposite flange.
Impact	Sudden application of a force in a very short period of time. An axe breaks through many fibres of wood at once by a "chopping" action.	How do you suppose a punch-press gets the job done?
Peening	Hardening by repeated "pounding" of the metal	This very old metallurgical technique has been "re-discovered" – we now know that we can extend the life of a steel girder bridge by "peening" these structural members.
Shear	To "chop off", breaking many fibres at once. Shear stresses tend to "cut" the fibres of a material.	Scissors cut fibres one at a time, rather than trying to break many fibres in a single action. A metal shear is a machine that combines the progressive slicing effect of scissors with the sudden "chopping" of an axe.

Keyword / Concept	An Initial Definition	Some "Things to Think About" Regarding Design and Fabrication Using Materials in General
Steel	A ductile ferrous material, prone to oxidation (rusting) if unprotected. Very strong, workable and useful. A common material for many products and structures.	Hot-rolled steel is harder than cold-rolled mild steel and is used for springs, cutting tools, dies etc. Higher-carbon content generally gives greater strength and hardness.
Strain-Hardening	Hardening of metal by putting it through several cycles of modest bending or stressing.	But if you bend too far too many times, you could completely break the metal.
Strength	A material's resistance to mechanical deformation due to tension, compression, torsion and other induced stresses.	Refer also to your work on "Structures". There are many different "measures" of strength, especially for anisotropic materials such as wood with its 3 axes – longitudinal, radial and tangential.
Yield Strength *	The yield strength is often determined to be the stress when a residual strain of 0.002 is not recovered when the load is removed – ie the material has started to "go plastic"	Deformations that are temporary due to the elasticity of the material are called *elastic*. When the stress is removed the member will return to its original shape. Deformations that 'stay' because the yield strength of the material has been exceeded by the loads are called *plastic*.
Ultimate Strength *	The stress when the material fails to sustain the load – fracture or breaking will occur.	When felling, the fibres on the tension side of a beech or ash tree will stretch surprisingly far before breakage finally occurs. In contrast, a white pine tree will very suddenly snap with very little lean of the tree when felling.
Young's Modulus, E *	Also known as Modulus of Elasticity. E is a property of a material -- how much it will change in shape or length under a given stress. $E = \frac{Stress}{Strain} =$ a constant for a given material in the material's elastic range. Stress is in units of force per unit area of the member's cross-section. Strain is essentially "stretch" or change in length -- either shorter in the case of compression or longer in the case of tension. Strain has no units.	 **Typical Stress-Strain curve in a tensile test of a low-carbon steel.** E is the slope of the straight-line "proportional" portion.
Rupture Strength *	aka Modulus of Rupture Relates to the maximum load in bending	Wood's Achilles heel is bending. So this figure is a good measure of a wood's strength and is often used in general cases.

Keyword / Concept	An Initial Definition	Some "Things to Think About" Regarding Design and Fabrication Using Materials in General
Tensile Strength *	The ultimate strength determined in a tensile test – pulling on a sample of the material. 	Wood is extremely strong in pure tension when the "pull" is parallel to the grain – ie along the length of a log. Wood will seldom fail in tension parallel to the grain. In this loading – such as in the bottom chord of a roof truss -- failure is much more likely to occur at a joint where a fastener could rip through to the end of the member. For example putting a screw into the end grain of a board such as pine will easily pull straight out. The Rupture Strength is typically used for most woods, since it is more conservative than tensile strength parallel to grain. If one were to pull from opposite sides of a short log to fracture, this would be tensile strength perpendicular to the grain -- radially. Sometimes this testing is done on a sample taken from one side of the log, but the pull is tangential to the growth rings. As such the tensile strength of a wood perpendicular to the grain is often an average of radial and tangential tests. Elm, with its highly convoluted grain patterns, has a relatively high value of tensile strength perpendicular to grain – ask someone who has ever split elm for firewood! *The illustration on left is from half-way up a 200 year old Frontenac County sugar maple that came down in a tornado in 2006. Note how the trunk grows around and repairs the wounds left by broken branches.*
Compressive Strength *	The maximum stress when tested to failure in compression.	When tested parallel to the grain, the wood member under test must not have a slenderness ratio greater than 11. Since buckling is somewhat "*compression gone wild*" and since a wood member typically fails in bending, wood columns and other members in compression such as the top chord in a roof truss must be laterally supported. Compression perpendicular to the wood grain is "crushing" across the grain. Oak is a very good bearing surface to support a concentrated load such as a post.

Table 1: Key Concepts

1.4 Properties of Common Southern Ontario Woods

Species	Dried Weight lb/ft^3	Janka Hardness lb_f	Modulus of Rupture lb_f/in^2	Elastic Modulus lb_f/in^2	S_c Crushing Strength* lb_f/in^2	Class / Comments
Eastern White Pine	25	380	8,600	1,240,000	4,800	Softwood, the general construction species of the 19th c in southern Ontario.
Balsam Poplar	23	300	6,800	1,100,000	4,020	Hardwood, but quite weak and softer than some softwoods. Grows quickly. Goes punky very quickly when dead and standing

Species	Dried Weight lb/ft^3	Janka Hardness lb_f	Modulus of Rupture lb_f/in^2	Elastic Modulus lb_f/in^2	S_c Crushing Strength* lb_f/in^2	Class / Comments
Eastern Hemlock	28	500	8,900	1,200,000	5,410	Softwood – harder than some hardwoods Very tough – used as planks on barn drive floors
White Ash	42	1,320	15,000	1,740,000	7,410	Hardwood – used for baseball bats
Black Cherry	35	950	12,300	1,490,000	7,110	Hardwood
Sugar “Hard” Maple	44	1,450	15,800	1,830,000	7,830	Hardwood
American Beech	45	1,300	14,900	1,720,000	7,410	Hardwood Now dying all over eastern Ontario
Basswood	26	410	8,700	1,460,000	4,730	Hardwood but softer than the softwood Hemlock
Black Locust	48	1,700	19,400	2,050,000	10,200	Hardwood Highly resistant to decay Extremely tough – used as pegs in ship-building
Eastern White Cedar	22	320	6,500	800,000	3,960	Softwood Highly resistant to decay; lightest Canadian wood Used as rail fencing, posts and roofing shingles

* Compression parallel to the grain

Source: http://www.wood-database.com, but the data was probably gathered from technical associations and their authorized testing laboratories. Some of this data seems to be the same as the data in Forest Products Laboratory General Technical Report FPL-GTR-113 “*Wood Handbook: Wood as an Engineering Material*” where the testing was done on samples at 12% moisture content.

Table 2: Properties of Common Woods

1.5 Variables in the Strength of a Material -- Uncertainties

1.5.1 General Properties of a Material

As discussed in Table 1 above *.

1.5.2 Iron / Steel

The yield strength of a standard commercial steel can be taken as 30,000 lb/in^2.

Steel comes in many grades, each with its own strength and other properties. The actual strength properties of a piece of steel, even with our carefully controlled modern processing methods, can differ from the applicable published nominal values as determined from exhaustive experiments. Among the most significant factors are heat treatment of the piece and peening.

Treatments such as these can:

- induce compressive stresses where additional hardness will be helpful;
- reduce tensile stresses where they may be detrimental to performance in an application
- promote helpful dislocations in the macroscopic crystal structure.

1.5.3 Wood

Wood is *anisotropic* as opposed to *isotropic*. An isotropic material has properties that are identical regardless of the direction in which they are measured. Because of a tree's cell structure and growth pattern, wood's strength properties depend on the direction in which they are measured. In particular, wood is *orthotropic*. Wood's strength properties depend on whether the property was measured:

- Parallel to the grain – the longitudinal axis, ie "up and down the tree"
- Radially across the grain – perpendicular to the growth rings, ie going outward from the centre of the tree
- Tangential to the growth rings

As such it is important to be clear on which direction within the wood that stresses are being generated.

Refer to Table 1 above * for some limited details.

There is a great abundance of variables in the strength and other properties of wood, including:

- Species of tree
- Growing conditions of the tree
- Location within the tree from which the piece of wood came (such as board cut from a cooked log), especially:
    - sapwood with waning (too close to the bark)
    - juvenile wood, nearest the pith of the trunk (ie wood in the first dozen or so growth rings of the tree's life)
- Residual stresses which come to bear when the piece of wood is cut from the timber (eg a trunk that had been leaning sideways – ie bending -- throughout its life)
- Stresses induced by kiln-drying and heat-treatment (kiln temperatures above 165^{o}F should generally be avoided)
- Moisture content (and associated shrinkage)
- Percentage of "late wood" (slower growing and denser) in the piece
- Grain angle (slope of grain)
- Large knots (both size and location) (knots should be small and located in the middle third of the piece's width)
- Other defects such as shakes (cracks along an annual rings), checks (cracks perpendicular to annual rings)
- Sudden changes in cross-section
- Insect damage

  - **Note**: In general, wood for use in a structure will be rejected if it shows evidence of insect boring (even though the heat treatment process will have killed all stages of insect life)

1.6 Other Uncertainties in a Structure

In addition to uncertainties respecting a material itself (see previous sub-section), other uncertainties enter the picture as soon as you put a material into a structure.

- Geometry of the member itself, for example:
  - thickness, depth, length of the member
  - the shape that the material is made to take – eg angle, channel, I-Beam, H-Beam etc.
  - how that shape was "put together" (rolling, forming, bending, welding, riveting, gluing etc.)
  - slenderness of the shape (in a very slender shape, an otherwise strong member in tension will instantly buckle when subject to only a very modest compression loading)
- Geometric relationship between the member of the subject material and other parts of the structure, for example:
  - How the member is oriented within the structure (eg a beam's resistance to bending increases by a factor of 8 if its depth is doubled – ie rotate a 2x4 by 90^o for 8 times the strength)
  - Size of and torque on and type of fasteners (fasteners should be torqued to put significant friction and compression between the two parts)
  - Proximity of fasteners to edges of the member – if insufficiently torqued, a fastener can rip apart the member at the joint
  - Bracing – adding triangles to corners of the structure
  - Stiffeners – similar to bracing but used to prevent buckling of a slender section
  - Lateral support for axially loaded columns in compression
- Loading Variables
  - Loads can be static (generally fixed and predictable) or dynamic (in motion)
  - Point loading
  - Uniformly distributed loading
  - Unpredictable loading made up of many point loads for example
  - Wind, snow, thermally-induced
  - The length of time under load
    - Cyclical, fatigue, creep
  - Chemical treatments
  - Environmental effects that can have an effect on strength properties of the material (including decay and insect / fungal damage in the case of wood)

Because of the multitude of variables (natural and application-related) affecting the strength of wood in construction, standards include a complex assortment of reduction factors and coefficients of variation which effectively reduce the allowable stress (of a particular kind) in a given situation. In addition to calculations involving reduction factors, a factor of safety must also be applied to deal with less quantifiable uncertainties.

2 An Over-Simplified Calculation for a Wood Column in Compression

Warning: The following discussion is over-simplified for the purposes of this high school course. The correct procedures and factors may be found in Canadian National Standard CAN/CSA 086-01 "*Engineering Design in Wood*".

Consider a vertical rough sawn white pine column 10 feet high, 2 inches thick and 4 inches wide. There are lateral supports or braces every 2.0 feet along its length, both in the plane of the column's thickness and in the plane of the width. What is the maximum vertical load that should be applied to the top of the column?

Pass One – Insufficient Solution!

From Table 2 above, the compressive strength parallel to the grain of white pine is:
$S_c = 4{,}800\ lb/in^2$.
Immediately applying a factor of safety of 10 gives a maximum allowable stress in compression of:
$S_{ac} = 480\ lb/in^2$
For a member that is 2" x 4" in cross-section, this suggests an allowable compression load of:
$L_{ac} = S_{ac}\ x\ A$
$L_{ac} = S_{ac}\ x\ 2\ x\ 4 = 480\ lb/in^2\ x\ 8\ in^2 = 3{,}840$ lb force

However, the column is very slender. If loaded in compression without lateral supports, this rough-sawn pine 2 x 4 will certainly buckle, even considering the use of a safety factor of 10 right off the top. The four lateral supports are added to prevent buckling, but there are calculations to make to reduce the allowable stress in the wood fibres, as follows.

Pass Two – Consider Reductions Due to the Slenderness

To prevent buckling of the slender rough-sawn 2x4, lateral supports were provided every 24 inches of height along both faces. The new unsupported length of 24" is called *l*. This gives two slenderness ratios:
$l_2/d_2 = 24 / 2 = 12$ where d_2 is the 2 inch thickness of the column
$l_4/d_4 = 24 / 4 = 6$ where d_4 is the 4 inch width of the column – less slender, thus less risky

$l_2/d_2 = 24 / 2 = 12$ is the more unstable slenderness ratio, so the solution continues accordingly

Note: The following solution is based on some information in J.H. Whitaker, *Agricultural Buildings and Structures* which was evidently based on US codes as of 1979. The solution assumes pin-hinged column ends. If one or both of the column ends was fixed against rotation, additional calculations would be involved. If the loading also included a bending component (such as in the case of a vertical load onto the peak of a tripod), additional calculations would also be involved. Three cases are involved depending on the slenderness ratio and the stiffness of the wood, a function of Young's Modulus E for the material. The result is the reduced allowable stress S_{rac}.

Case One:
For short columns where l/d < 11, the governing formula is:
$S_{rac} = S_{ac}$ In other words, for very short columns, the column will not buckle and no reduction is required. Thus a 4 x 4 white pine column (with *l*/d = 6) can take a full load of 7680 lb if it is supported laterally every 24 inches.

Case Two: (our situation, where the most critical slenderness ratio is l/d = 12)
For columns of intermediate slenderness ratio where 11 < l/d < K
Where
$K = 0.671 \sqrt{(E/ S_{ac})}$
For white pine
$K = 0.671 \sqrt{(1,240,000 / 480)}$
$K = 0.671 \sqrt{(2583)}$
$K = 0.671 \times 50.8$
$K = 34$
Since, in this situation the most critical slenderness ratio is l/d = 12 < K
Reduced allowable stress S_{rac} is given by
$S_{rac} = S_{ac} (1-1/3(l/d/k)^4)$
$S_{rac} = 480 (1-1/3(12/34)^4)$
$S_{rac} = 480 (1-1/3(.353)^4)$
$S_{rac} = 480 (1-1/3(.0155))$
$S_{rac} = 480 (1-.00517)$
$S_{rac} = 480 (.9948)$
$S_{rac} = 477 \text{ lb/in}^2$
The reduction in allowable compression stress is very modest because *l*/d is so close to 11.
The reduced allowable load on this 2 x 4 inch member is:
$L_{rac} = S_{rac} \times 2 \times 4 = 477 \text{ lb/in}^2 \times 8 \text{ in}^2 = 3,816$ lb force

Case Three:
For long columns where *l*/d ≥ k:
Reduced allowable stress, S_{rac} is given by a third formula:
$S_{rac} = 0.3 E / (l/d)^2)$

If, in our problem, the cross-section of the column is changed from 2 x 4 inches to 1 x 4 inches and if the unsupported length is changed from 24 to 36 inches the slenderness ratio would be:
l/d = 36 / 1 = 36
In this case where *l*/d = 36 > k, the third formula applies
$S_{rac} = 0.3 (1,240,000) / (36)^2)$
$S_{rac} = 0.3 (1,240,000) / (1296))$
$S_{rac} = 0.3 (957)$
$S_{rac} = 287 \text{ lb/in}^2$
The reduced allowable load on this 1 x 4 inch member is:
$L_{rac} = S_{rac} \times 1 \times 4 = 287 \text{ lb/in}^2 \times 4 \text{ in}^2 = 1,148$ lb force

Note: Obviously, the lateral supports must be large enough in cross-section and short enough in length that they themselves will not buckle.

3 The Net – Bottom Line

The professional designer must have a profound understanding of the system and the conditions in which the system is most likely to operate. The system includes materials and how materials are processed; how materials are transformed into geometrical shapes / configurations; and how shapes are connected together into a sub-system.

Where there is uncertainty, the design professional must:

- Obtain additional relevant data
- Use appropriate and proven tools to properly reduce and understand the new data
- Re-visit assumptions that were previously made
- Set limitations around acceptable applications of the design (eg *"for a span not greater than …"*)
- Set an acceptable safety factor that reflects the remaining uncertainties – for wood, a safety factor in the range of 10 seems appropriate in the general case
- Regardless, calculate and apply allowable stress reduction factors that are prescribed in industry standards.

4 Self and Peer Assessment

Inputs / Knowledge / Understanding That I Still Need or Connections that I Want to Make For This Unit: (give each a #)

Assessor's Name and Notes: (Peer Assessor must "make you think". Give the student one clue that will help him or her get a better mark.)

Endnotes

[i] S. Timoshenko, *A History of Strength of Materials*, 1953. The two preceding quotes are also from this book.