Problem-Solving: Master the Fundamentals First

Concepts / Skills: Structure, System, Safety, Analysis, Synthesis, Problem-Solving, Deriving New Concepts from Fundamental Concepts

Grade 12 Technological Design

CONTENTS

1 Expectations: The Student will:

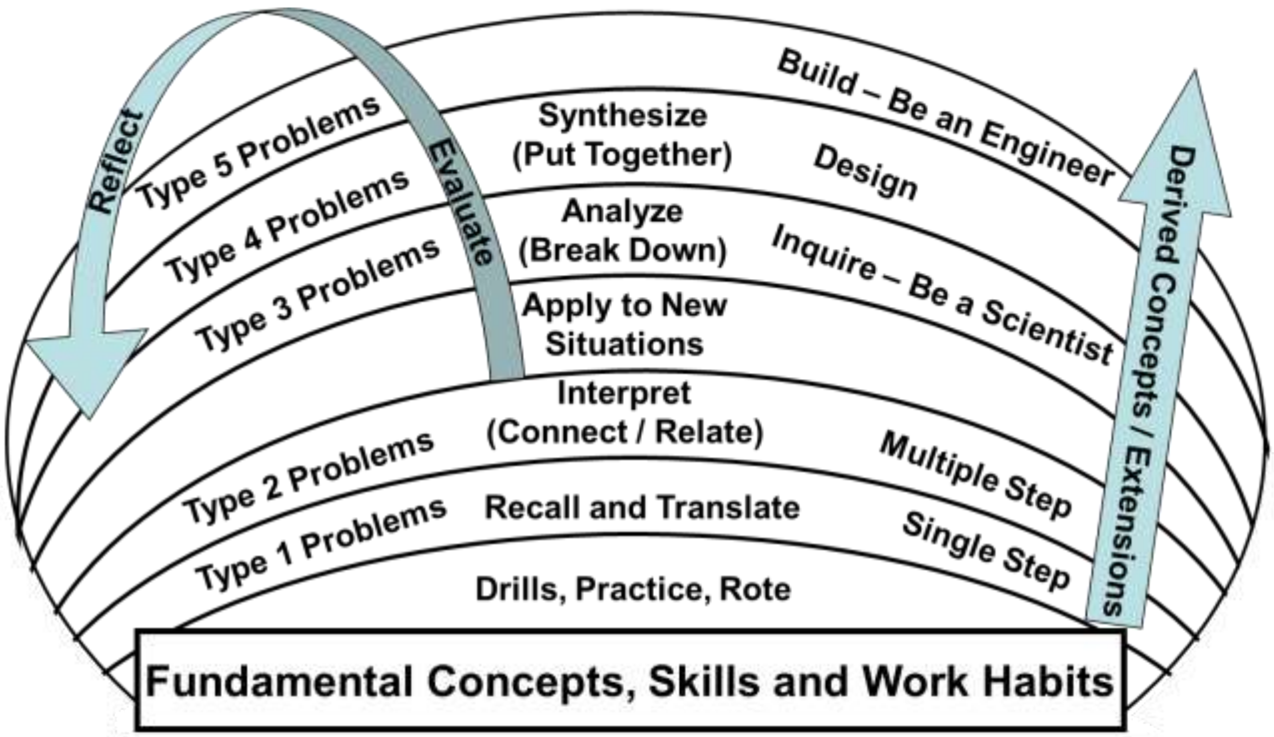
1) **Problem-Solving; Numeracy; Application of concepts and tools in a new situation**:
   a) Distinguish between Black Box and White Box analysis strategies
   b) Derive new concepts from fundamental concepts
   c) Practice the kinds of thinking that an innovator, such as Thomas Telford (1757-1834), might have used in the development of new technology for the industrial revolution's infrastructure
   d) Begin to extend knowledge of these fundamental concepts from one application (eg truss kind of beam bridge) to a more complex application (eg suspension bridge or a special creative architectural feature)

2 Problem Solving – An Overview

The visual model below shows seven core thinking skills that you can use to derive new concepts and solve increasingly more complex problems.

There are five general types of problems to solve in this problem-solving model:

1. ***Simple Operation***: Single Step (common in math class; eg. Do I multiply or divide?)
2. ***Exploration***: Multiple Step / Linear System (common in math class)
3. ***Process***: Inquiry-Based (often connecting math and science or math and history)
4. ***Design***: Open-Ended Data-Gathering (often bridging math, science and tech studies)
5. ***Fabrication***: Open-Ended Inquiry-Design-Build (eg. Technological Design)



Think About It --

--Search this document for the word "engineer"

--But this document seems to be about bridges!

--Or is this document really simply about sound approaches to solving problems in any field?

3 Review -- Ideas / Concepts and Connections

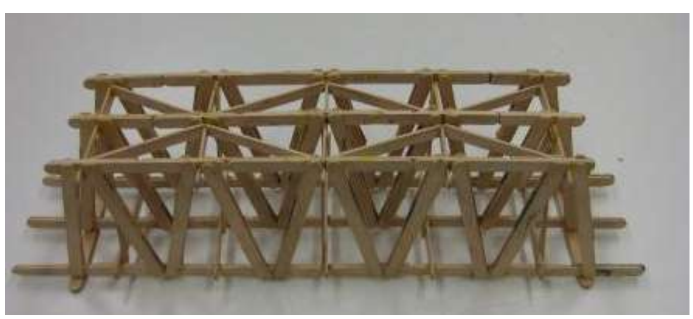


Figure 1: Truss

1) What is so "smart" about a truss type of beam bridge design?
   a) A truss bridge simply puts the beam's material in the locations where it will be of maximum benefit when subjected to bending loads. For a typical beam bridge supported at the extreme ends with the downward-directed load on top, maximum material at the top of the bridge (the top stringer of the truss) is required to take compression stresses and maximum material at the bottom (the bottom stringer of the truss) is required to take the tension stresses. This very smart beam bridge design strategy – a truss – eliminates much dead load and saves material.

2) In a truss, what is the purpose of the diagonal members that connect the bottom stringer and top stringer together?
   a) These pieces which complete the "triangles" simply carry the load from top stringer to bottom stringer and then out to the bearing supports at the two ends of the bridge.

3) In the context of bridges, interpret your own meaning of the concept "structure" using some other keywords that you have come across in your high school studies. Write an "equation" if you like – to make your interpretation as clear to you as possible. In other words, find your own way to define "structure" in the context of your role as a designer of some new architectural element for example.
   a) **Structure** = **Geometry** + **Planned Response** (ie to loads by distributing the forces through the key strategically-placed members).

4 Establish a Framework / Strategy and Know the Fundamentals

To be a good designer takes more than just creative thinking. Good design also requires you to be a critical thinker – how does this particular system work anyway? The depth of your analysis of a system will determine your true level of understanding. If you don't understand the system, you can't properly design for it or fully solve problems related to it.

4.1 Black Box Modelling – Starts With Really Basic Laws and Principles

Looking at something as a "black box" is one early step that you can take to greatly simplify a problem. There are inputs to a black box and there are outputs from the black box. You build your understanding of the system around those inputs and outputs. You don't care what is inside the black box. And you certainly don't care how the black box "works". In other words, you don't care about the "process" that is going on inside the black box. This strategy for understanding allows you to focus on the parameters of the problem (inputs) and what it is that you generally want to achieve (outputs) with your system. You ignore the details of how it all works. Again, the intent of this black box simplification strategy is to increase your level of "understanding" of the system to better prepare you to creatively design related solutions.

For example, suppose you want to build a long and light yet strong model bridge. Desired strength is the output – what you want to achieve. Length, an amount of construction material (weight) and the loads are inputs to the system. Using a black box methodology, you can conduct a series of experiments to determine relationships between the outputs and the inputs – how does strength depend on the material for example.

Thomas Telford, designer of (arguably) the world's first modern suspension bridge, was a master at this approach to *"achieving an understanding".*

4.1.1 Simple Sample Analysis Problem: First Model a Rigid Body as a Big Particle

These three members are pinned together using a simple peg or dowel at the joints. A fixed support is at the left end while a roller supports the right end. Which members are in compression and which are in tension? Label the three members C or T. Don't guess about this -- think it through by reasoning out what is happening as the structure responds to the load F.

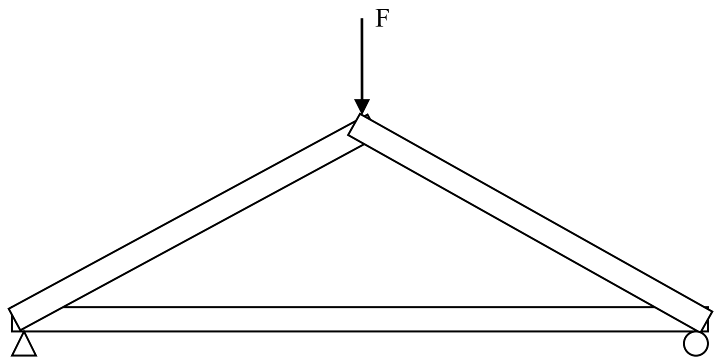


Figure 2: "Solve" this Structure – ie Come to an "Understanding" of It

First, we make some basic assumptions. The problem statement says nothing about whether or not the object is moving. So we assume it isn`t – it is neither moving left nor right nor up nor down. We can also assume that it is not spinning around. Thus, we are assuming that this "particle" is in a state of equilibrium.

By using our notion of Black Box, we can first view this structure as a simple (but clumsy-looking) particle. Sure this particle is large and composed of three connected members, but our strategy in some respects is to view the forces acting on it – their magnitude and direction of application – in relation to a point that represents the structure as a particle. At this stage, we don't care "how the structure works".

We start with some basic laws. The structure is a triangle, so it must have a centroid. Further, by way of another assumption, the structure appears to be an isosceles triangle, so it must have a line of symmetry passing vertically down through the point of application of force F. This line of symmetry must pass through the centroid. We can generally treat the mass of this structure as though the mass was all concentrated at the centroid. But we still keep in mind the finite size of the structure.

Now draw the structure again, but represent the 2 supports as lines of force (or force vectors). (As in math class – "Re-Represent" the problem to make it easier to understand!) Since there is no horizontal component of F, there can be no horizontal forces acting on the structure. The structure is neither moving left nor right. So draw the two vertical reaction forces R_l and R_r (Reaction left and Reaction right). Refer to the free body diagram below. Because the structure was sitting on supports, it is neither moving up nor down. We can then say that:

$F = R_l + R_r$	(not moving up / down)
$d_l = dr$	(assumption of symmetry)
$R_l \times d_l = R_r \times d_r$	(not rotating or spinning about the centroid (shown as ●) (law of the lever)
$R_l = R_r$	(divide both sides by d_r)

We can now point out that $F = 2R_l$
$F - 2R_l = 0$
F and ($- 2R_l$) are equal and opposite forces (force vectors).

Our concern in this black box modelling simply relates to the rigid triangular shape which we treated more or less as a big particle. It is neither moving up nor down, neither left nor right and it is not spinning. (And it is not moving in or out of the plane of the page – another assumption!.) The result or resultant of the three forces acting on the object is zero. The object is in equilibrium. Note that, to deal with rotation of the structure, we did take into account the distance between the points of application of the two reaction forces and the centroid. In this portion of the solution we viewed the structure as a rigid body, not as a simple infinitely small particle.

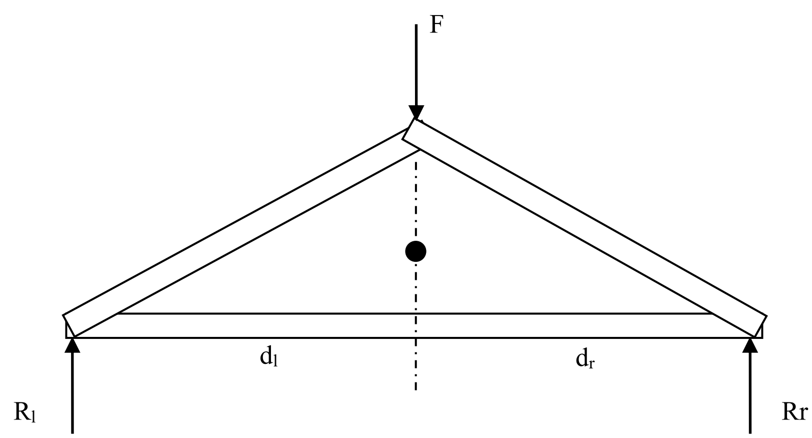


Figure 3: Free Body Diagram

So far, we have looked at the state of equilibrium of the object.

4.2 White Box Modelling -- What's Going on Inside?

In contrast, a "white box" model is what you have when you want to look inside to see what is going on. You usually need a good strategy to be successful when looking inside at the details. And you typically need some of the science of what is inside the box. In the early 19th century neither Thomas Telford nor his peers had much (if any) of the materials science and strength of materials knowledge that we have today. We will, shortly, continue the above example of the triangular frame analysis.

4.2.1 Deriving New Concepts from Fundamental Concepts

But first, consider how we can improve our learning by building on certain fundamental concepts in the study of technology. Our immediate goal is to build understanding of structures in particular. We will derive new concepts in the study of structures from those fundamentals that we should have already more or less "mastered". People have defined new concepts through many degrees or levels of derivation. It is time to start proving to yourself that you 'buy into' these concepts – prove them yourself. ***In addition, you should start to derive your own creative new, even simplified, ways of looking at concepts such as strength and structure in the table below. The following methodology will help you improve your learning in any area – not just in relation to structures, one of the 13 fundamental concepts of technology. We can be absolutely certain that Thomas Telford practiced these approaches to problem-solving and opportunity-achieving.***

The table below is sorted alphabetically by the Keyword / Concept column – not particularly helpful for your learning. You already know about the 13 fundamental concepts of technology. You already know about certain fundamental laws in physical science. (We will limit this to Newtonian mechanics – the everyday world that we can see and touch and hear. Relativity and systems where objects are moving at close to the speed of light are not under our consideration.) You also already know about some principles in relation to how you can assess the physical world using tools of mathematics. And there are also some general "good practices" to follow such as standards, the design process etc. So, sort the following concepts by these categories: **T (Technology), P (Physics), M (Math) and G (General)** in order to help your learning. The Tree Number column can be used (by you) to help you start to build a hierarchical tree of a set of concepts. After you have sorted out your hierarchy in the table, sketch out a version 1 of your visual tree. Keep each tree small and focused. A sample tree sort order is shown in the table – sort by the Tree column numerically.

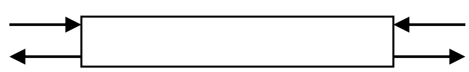
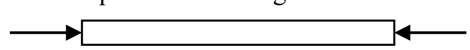
Note that, in the list below, the only absolute fundamental concepts used in Newtonian mechanics are space, mass and time. These 3 concepts cannot be truly defined. However, to succeed in the study of the physical world, these 3 concepts must be understood through intuition, experience and experimentation – the key strategies employed by Thomas Telford. Then these 3 concepts can be appreciated as a valid frame of reference upon which to build further understanding.

In addition, Force is shown in the table below to be both "derived" and a top-level concept in our hierarchical tree – level 4, making it almost as fundamental as space, time and mass. How can this be? Because, as mentioned above, space, mass and time cannot be truly defined, we must come to understand these 3 concepts by observing objects in motion, ie by experimenting. Force is the concept or idea which physicists invented (or derived) to link space, mass and time together. Of course, this is where Newton`s Laws of Motion come in.

EXERCISE – MAKING CONNECTIONS

In column 5 on the far right, insert your own thoughts on how this particular concept (in column 3) relates to your particular design problem – whatever your design problem

happens to be. You could be designing a chair or a ladder or a new kind of shoe. Several examples are given in column 5 below – if the example is not relevant to your application design problem – delete it and enter your own.

1 **Tree #**	**2** **Sort Group**	**3** **Keyword / Concept**	**4** **Derivation or an Initial Definition**	**5** **Some Issues to Think About In the Context of Product Development or Your Situation**
4.7	P	Bending Moment	**Derived**: A bending moment is generally the result of a force applied at a distance – ie 'coupled' to sections of the member via the structure. In the strict sense, a 'couple' is formed by 2 forces having the same magnitude, opposite directions and parallel lines of action. The moment of a couple tends to make the object or section of the object rotate – ie bend in the case of a section. Bending is a loading configuration that puts part of a member in tension and part of the member in compression. 	Resistance to bending is proportional to the cube of the depth of the member. The fibres of the material at the neutral axis -- the horizontal line at the midpoint of the diagram -- are under zero stress. Thus, these fibres in the centre of the member are of virtually no use in resisting bending -- they are simply dead weight or dead load.
4.5	P	Compression	**Derived:** One of many kinds of forces A Push force -- the member gets shorter. Particles pushed closer together 	
4	P	Force	**Derived** from a consideration of the measured mass of an object and the changes in its behaviour in space over time – ie the motion of the object (how its velocity varies over time). A force is something that causes a change. There is typically a resistance to such a change. In the Newtonian sense, a force represents the action of one body on another body. A force has a magnitude, direction and point of application.	Use a free body diagram to visualize the various forces acting on a part.
1.1	M	Geometry	**Rules / Tools -- Math** Physical arrangement -- addressing form and shape. Geometry is the fundamental mathematical way of looking at the fundamental physics concept of **space**. In other words, geometry is a set of tools to help us understand space, measured in units of length.	Careful geometrical arrangements add strength to a structure, while minimizing the material costs. Putting material where it is not needed is wasteful and just adds dead load. A triangle is a very stable arrangement of three pieces. Careful attention to geometry allows designers to design a more efficient structure.
3	P	Mass	**Fundamental Concept** The idea of mass is used to study the behavior and properties of bodies. This is typically done by experimentation.	

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			If mass is considered fundamental, then **force** is derived from a consideration of mass, time and space.	
3.1	T	Material	**Derived** The substance or information from which the structure is made. This concept, in the context of technology development, is not limited strictly to physical objects having a finite mass – hence the reference to "information". Studying physical materials is a way of looking into the more fundamental concept of **mass** in a very wide variety of ways (ie properties of a material).	Some wood types are very tough and strong such as oak, ironwood and ash. Wood is strong when loaded in pure tension. Wood is strong in pure compression when loaded across the grain (ie, trying to crush the wood). But a long slender wooden member will buckle easily when in pure compression along the grain. Wood has some other particular concerns. When loaded in bending, the tension that is introduced is, in a manner of speaking, 'magnified' due to the pivot point -- like in a lever. So a long slender wood member is relatively weak in bending -- but that same member is often incredibly strong in pure tension. A knot in a wood member is a "discontinuity" that severely impacts strength in bending and tension. In a pinned connection the pin may split the wood if the pin is too close to the end of the member.
4.1	P	Newton`s First Law	**Fundamental Law** If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant velocity in a straight line (if originally in motion.)	
4.2	P	Newton`s Third Law	**Fundamental Law** For every action there is an equal and opposite reaction.	These two equal and opposite forces also have the same line of action
4.3	P	Parallelogram Law for the Addition of Forces	**Fundamental Law** Two forces acting on a particle may be replaced by a single force, their 'resultant'. The resultant is obtained by drawing the diagonal of the parallelogram which has sides equal to the magnitude of the two forces. The resultant has both magnitude and direction. The point of application of the resultant is still the particle.	
4.8	P	Pressure	**Derived** Intensity of force eg force per unit area	Typically referred to as "stress" in structural analysis and design.
4.4	P	Principle of Transmissib-ility	**Fundamental Law** The conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and the same direction but acting at a different point, provided that the two forces have the same line of action	
3.2	P	Properties	**Derived** Characteristics of an object Every classification of object in the world	Structural materials must be selected on the basis of carefully controlled and well-understood properties such as hardness,

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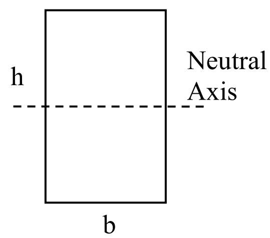
			generally has a number of characteristics such as material, size and shape. These are called "Properties" of that class of object.	ductility, strength and resistance to corrosion.
1.4	M	Second Moment of Area about the Neutral Axis (I)	**Derived** For this rectangular beam section where h is the height and b is the width: $I = \frac{bh^3}{12}$ 	
3.5	P	Simple Beam Bending Formula	**Derived** $s = \frac{My}{I}$ Where: --- s is the value of the stress "felt by" the material at a particular point (at point y in the cross-section). The maximum stress that can be endured by a material is proportional to Young's Modulus, E. E is a property of that material. --- y is the distance up or down from the neutral axis of the beam. In a symmetrical section, this is the distance from the midpoint of the section. --- M is the bending moment to which the beam is being subjected at that point in the span of the beam. Bending moment is applied load times distance. A bending moment is one of the net results of loads acting on a beam. --- I is the second moment of area of the section about the neutral axis Note that if I increases, the stress decreases. The stress increases as the distance from the neutral axis increases.	Several assumptions were made in the derivation of this formula for stress due to bending, eg the beam is slender (length dimension is much greater than its depth dimension) and the beam's cross-section is uniform and the beam's material has consistent properties along its length. As in many other situations, the use of this formula is limited to such a range of permitted values.
1	P	Space	**Fundamental Concept** The position of a point is given by the three distances (its coordinates) from a system reference point (origin). Space is measured in units of length.	
5	G	Standard / Convention	**Rules / Tools** A set of rules or definitions. For instance, there is a standard length of a metre. There	Structural testing should always be done against standard procedures and test methods. For example, an elevation above sea level

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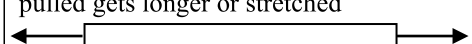
			are also standards of good practice on work sites. A convention could be as simple as deciding, in the case of opposites for example, how a value will be determined to be either positive or negative.	would be considered positive.
3.3	T	Strength	**Homegrown Derivation** The following is a home-grown and very simple derivation of a new concept from more fundamental concepts… Strength = Structure + Material	How can you build your understanding of strength? Start by digging deeper into the concepts of Structure and Material.
3.6	P	Strength of a Beam	**Derived** The strength of a beam is proportional to the second moment of area of the beam's cross-section about the beam's neutral axis. For a rectangular beam, if the height doubles, the second moment of area -- and hence the beam strength -- increases by 8 times (2 cubed).	
1.3	T	Structure	**Derived** The essential physical or conceptual parts of a product, process, or system, including the way in which the parts are constructed or organized. In the context of technology development, structure is not limited to a discussion of physical objects. **Homegrown Derivation** In the physical Newtonian world we could simplify the idea of designing a structure in a simple 'equation' such as: *Structure = Geometry + Planned Response to loads by distributing the forces through the key strategically-placed members.*	Deformations that are temporary due to the elasticity of the material are called elastic. Deformations that 'stay"because the yield strength of the material has been exceeded by the loads are called plastic. Build your understanding in way that helps you.
1.2	M	Symmetry	**Tool -- Math** The similarity of two parts of an object when divided by an imaginary line. Assuming symmetry simplifies analysis. Symmetry simplifies design. Building in symmetry simplifies troubleshooting, service and maintenance.	A symmetrical member is less likely to prematurely buckle than an asymmetrical member under pure compression stresses.
4.6	P	Tension	**Derived:** One of a number of types of forces. Tension is a Pull force – the member being pulled gets longer or stretched 	
2	P	Time	**Fundamental Concept** An event in the Newtonian world must be defined in terms of the three fundamental	

1 **Tree #**	**2** **Sort Group**	**3** **Keyword / Concept**	**4** **Derivation or an Initial Definition**	**5** **Some Issues to Think About In the Context of Product Development or Your Situation**
			and independent concepts, position in space, mass and time	
3.4	P	Young's Modulus, E	**Derived** E is a property of a material -- how much it will change in shape or length under a given stress. $E = \frac{Stress}{Strain}$ = a constant for a given material Strain is essentially "stretch" or change in length -- either shorter in the case of compression or longer in the case of tension. By way of simple example, the deflection of a spring is proportional to the load pulling on it.	

Table 1: Important Concepts About Structures

Think About It --

-After looking over the discussion of Free Body Diagrams below, where would you insert "Free Body Diagram" in the table above? What definition would you give for "Free Body Diagram"? What are a couple of "Issues to Think About" regarding "Free Body Diagram" – especially in the context of your design problem?

-Do exactly the same as above for "Trigonometry" – which you learned in grade 10 math.

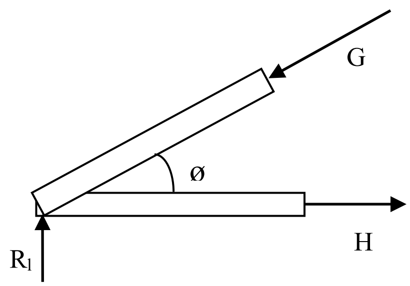
-Do exactly the same as above for the "Law of the Lever" – which you studied in elementary school science.

4.2.2 Structural Analysis (Continued)

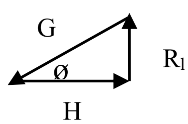
Now go back to the problem statement from the previous section (4.1) -- the three members of the triangular structure above are pinned together using a simple peg or dowel at the joints. We are asked to determine which members are in compression and which are in tension. (In the next lesson we make logical assumptions related to pin-connected joints. For now however the problem statement simply tells us that the members are in either compression or tension, so we can proceed without making further assumptions.)

The problem statement is asking us to come to an understanding of how the above 3-member structure works to support the load F. In other words, we are being asked to go inside the structure to determine precisely how the materials that make up the structure respond to the external forces F, R_l and R_r

To view the structure as a white box – to see what is going on inside those three slender members – we imagine that we actually cut right through the structure and look in detail at one of the portions. Let the force in the diagonal member be represented by force line (vector) G. Let the force acting in the horizontal member be represented by force line (vector) H. These 2 forces as well as R_l all act through the left support vertex. Equilibrium still applies and the resultant of the force acting through that vertex must be zero. It is neither moving nor rotating.



Draw a force diagram showing that the three forces acting on the point sum to zero and apply your grade 10 Trigonometry.



Sin ø = R_l / G

G = R_l / Sin ø

Tan ø = R_l / H

H = R_l / Tan ø

Force G is pushing (compression) downward and somewhat to the left at angle ø through the member to counterbalance a part of the R_l force which is pushing up. At the same time, force H is pulling (tension) rightward on its member to counterbalance that portion of force G which is pushing leftward.

Thus, we have shown that the diagonal member from the left support to upper right is in compression (being pushed), while the horizontal member is being pulled (in tension).

Thomas Telford would have probably been familiar with the triangle of forces approach discussed immediately above. This was probably the extent of what he could do with "white box modelling" -- ie looking "inside" the objects in order to understand their behaviour.

4.2.3 Another Technique – Extreme Cases

Note that one other problem-solving technique for solving the structure in Figure 2 is to take the problem to an extreme case. Suppose the horizontal member were suddenly removed -- what would happen? The two other members would suddenly flatten and the two supports would spread apart. So, the horizontal member must have been holding or pulling them together – hence the horizontal member is in tension. This could be viewed as an intuitive approach to solving this simple problem. And of course, this is the kind of thinking that Thomas Telford was so good at.

** The teacher will later demonstrate a technique for understanding an ideal simply supported beam bridge under a bending load. This involves many of the terms shown in Table 1 above.

5 Checking Understanding

- Do the exercise discussed in section "*Deriving New Concepts from Fundamental Concepts*"
- Do the Structures concepts quiz at http://thinkproblemsolving.org/login/index.php

6 Safety Reminder

Safety is the number one concern in bridge design / construction and this depends greatly on the strength of the bridge. People can die when a bridge fails structurally.

7 Self and Peer Assessment

Inputs / Knowledge / Understanding That I Still Need or Connections that I Want to Make For This Unit: (give each a #)

Assessor's Name and Notes: (Peer Assessor must "make you think". Give the student one clue that will help him or her get a better mark.)
